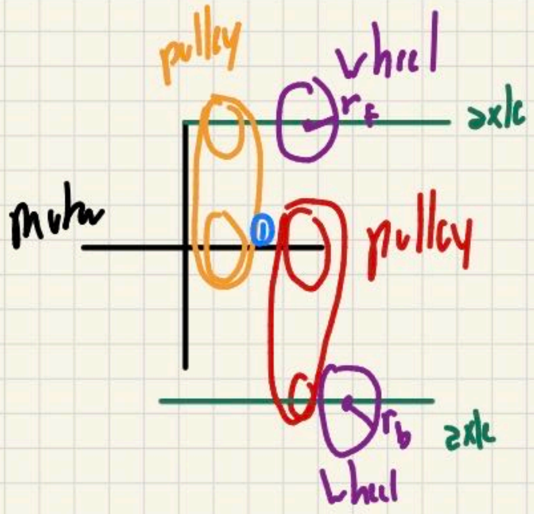
3VIb Bot Indirect (Rigid) Belt Drive Drivetrain Analysis

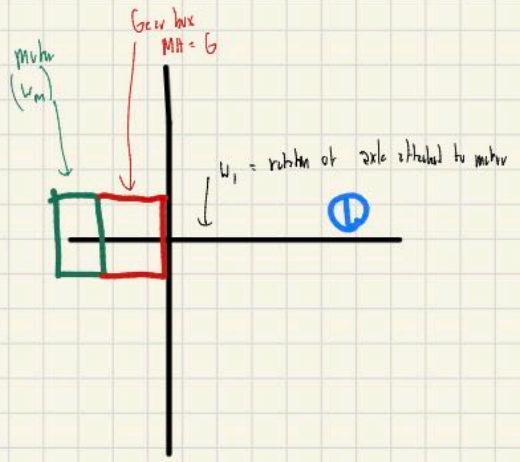
Kaden Dadabhoy

System:

front



Motor - 1



$$MA = \frac{V_m}{V_{out}} = \frac{r_{out}}{r_{in}} = \text{"}G\text{"} > \text{gear}$$

$$V_m = V_1$$

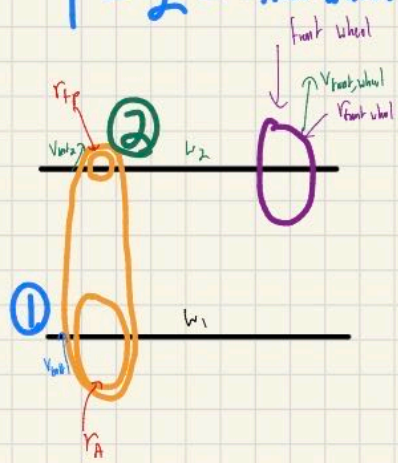
$$r_m w_m = r_1 w_1$$

$$w_1 = w_m \frac{r_m}{r_1}$$

$$\frac{r_{out}}{r_m} = \frac{r_1}{r_m} = MA$$

$$w_1 = \frac{1}{G} w_m$$

1 - 2 - $V_{front\,wheel}$



$$V_{belt_1} = V_{belt_2} = V_{belt}$$

$$V_{belt} = w_1 r_A$$

$$V_{belt} = w_2 r_{fp}$$

$$w_2 r_{fp} = w_1 r_A$$

$$w_2 = \frac{r_A}{r_{fp}} w_1$$

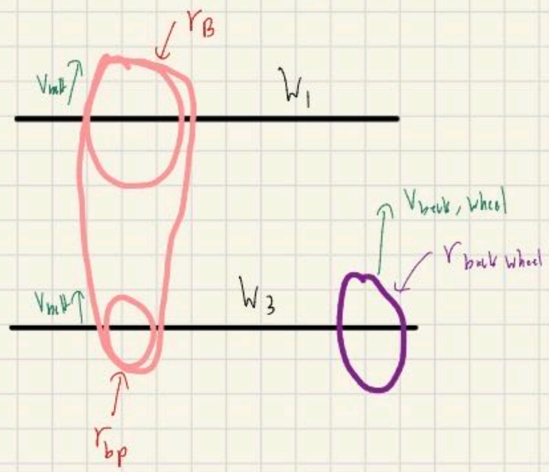
$$w_2 = \frac{1}{G} \frac{r_A}{r_{fp}} w_m$$

$$\rightarrow \frac{r_A}{r_{fp}} = \frac{\text{Teeth Motor Pulley (Driver)}}{\text{Teeth Forward Pulley (Driven)}} = \frac{1}{\text{Front Speed Ratio}} = \frac{1}{R_f}$$

$$w_2 = \frac{1}{G R_f} w_m$$

$$V_{front\,wheel} = w_2 r_{front\,wheel} = \frac{r_{front\,wheel}}{G R_f} w_m$$

1 - 3 - $V_{back\,wheel}$



$$V_{belt} = r_B w_1 = r_{bp} w_3$$

$$w_3 = \frac{r_B}{r_{bp}} w_1$$

$$w_3 = \frac{r_B}{r_{bp}} \left(\frac{1}{G} w_m \right)$$

$$\longrightarrow \frac{r_B}{r_{bp}} = \frac{\text{Teeth Motor Pulley (Driver)}}{\text{Teeth Motor Pulley (Driven)}} = \frac{1}{R_{back}}$$

$$w_3 = \frac{1}{G R_{back}} w_m$$

$$V_{back\,wheel} = r_{back\,wheel} w_3$$

$$V_{back\,wheel} = \frac{r_{back\,wheel}}{G R_{back}} w_m$$

So...

* We need $V_{back\ wheel} = V_{front\ wheel}$ in order to go straight... obviously

→ So:

$$\frac{r_{back\ wheel}}{R_{back}} \cdot \cancel{\frac{W_m}{G}} = \cancel{\frac{W_m}{G}} \cdot \frac{r_{front\ wheel}}{R_{front}}$$

$$\boxed{\frac{r_{back\ wheel}}{R_{back}} = \frac{r_{front\ wheel}}{R_{front}}}$$

* $R_{back} = \dfrac{\text{Teeth Back Pulley}}{\text{Teeth of corresponding Motor Pulley}}$

* $R_{front} = \dfrac{\text{Teeth Front Pulley}}{\text{Teeth of corresponding Motor Pulley}}$

→ Assuming this is satisfied:

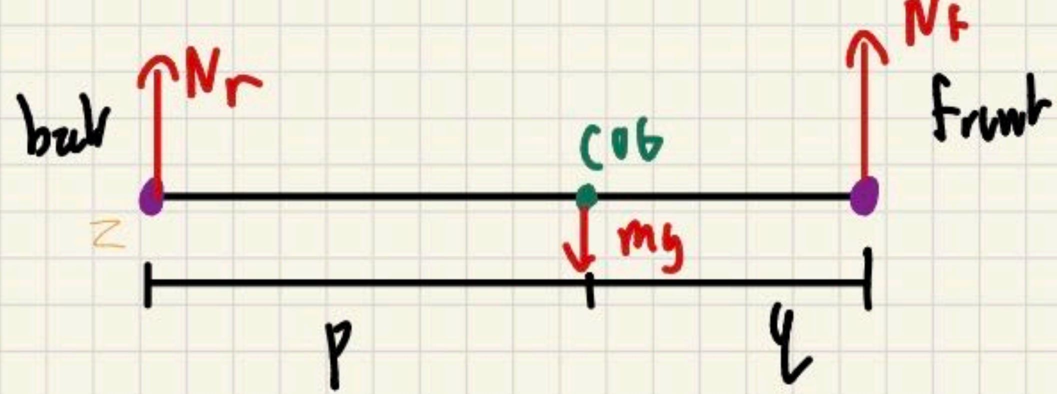
We can say:

- $V_{front} = V_{back} = V$
- $V = \dfrac{r_{front\ wheel}}{G R_f} W_m = \dfrac{r_{back\ wheel}}{G R_b} W_m$

→ $\boxed{\dfrac{r_{front\ wheel}}{G R_f} = \dfrac{r_{back\ wheel}}{G R_b} \equiv K_{sys}}$

* $\boxed{V = K_{sys} W_m}$

Statics of System;



$$M_z = 0 = N_f\,(p+q) - mg\,(p)$$

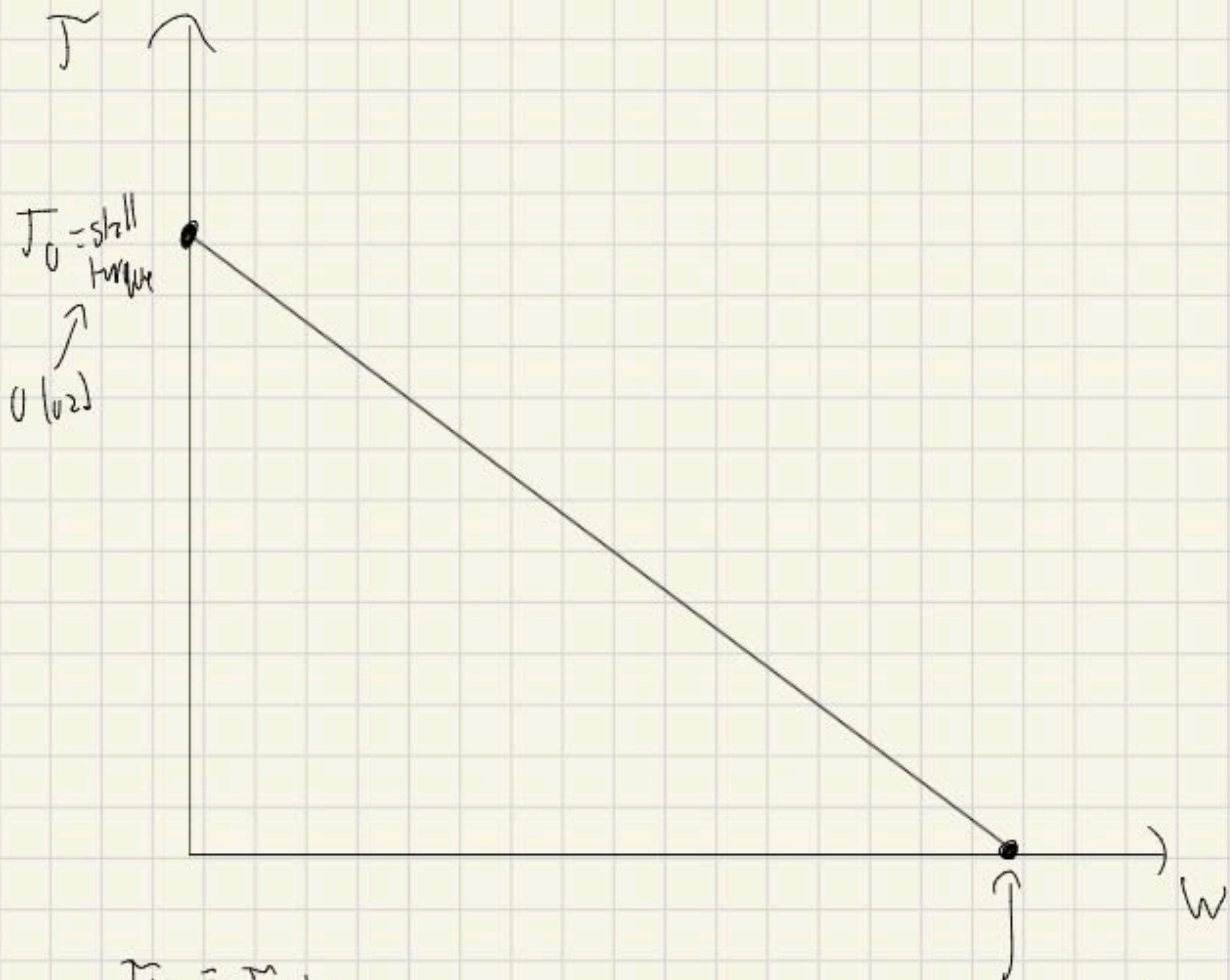
$$N_f = \frac{mg\,p}{p+q} = mg\left(\frac{p}{p+q}\right)$$

$$\Sigma F_y = 0 \Rightarrow N_f - mg + N_b = 0$$

$$N_b = mg\left(1 - \frac{p}{p+q}\right)$$

* Where mg = Weight of 1 side = $\frac{\text{Total Weight}}{2}$

DC Motor $\tau - \omega$ curve



ω_{NL} = no load

$\tau_m \equiv \tau_{motor}$
$\omega_m \equiv \omega_{motor}$

$$\boxed{\tau_{out}} \equiv \tau_m = -\frac{\tau_0}{\omega_{NL}}\omega_m + \tau_0 = \boxed{\tau_0\left(1 - \frac{\omega_m}{\omega_{NL}}\right)}$$

just renaming for later purposes

$$\tau_0 = \underbrace{I_{max} \cdot k_t}_{\text{motor properties}}$$

$$\omega_{NL} = \underbrace{\text{Voltage} \cdot K_V}_{\text{motor properties}}$$

Work Energy Conservation

Power in = Power out

$$\text{Power} = \tau \cdot w = \frac{d}{dt}\left(KE_{total}\right) \rightarrow \text{in some cases } = Fv$$

$$T_m w_m = \frac{d}{dt}\left(KE_{chassis} + KE_{front\ pulley} + KE_{rear\ pulley} + KE_{motor\ shaft}\right)$$

$$T_m w_m = \frac{d}{dt}\left(\frac{1}{2} m \underset{v = w_m k_{sys}}{v^2} + \frac{1}{2} I_{fp} \underset{w_f = \frac{w_m}{R_f}}{w_f^2} + \frac{1}{2} I_{rp} \underset{\frac{w_m}{R_r}}{w_r^2} + \frac{1}{2} I_m w_m^2\right)$$

$$T_m w_m = \frac{d}{dt}\left(\frac{1}{2} m k_{sys}^2 w_m^2 + \frac{1}{2} \frac{I_{fp}}{R_f^2} w_m^2 + \frac{1}{2} \frac{I_{rp}}{R_r^2} w_m^2 + \frac{1}{2} I_m w_m^2\right)$$

$$T_m w_m = \frac{1}{2} \underbrace{\left[m k_{sys}^2 + \frac{I_{fp}}{R_f^2} + \frac{I_{rp}}{R_r^2} + I_m\right]}_{I_{eq}} \frac{d}{dt}\left(w_m^2\right)$$

$$T_m w_m = I_{eq} w_m \alpha_m$$

$$\boxed{T_m = I_{eq} \alpha_m}$$

$$\rightarrow a = \alpha_m k_{sys}$$

$$\text{so } T_m = \frac{I_{eq} a}{k_{sys}}$$

$$\boxed{a = \frac{T_m k_{sys}}{I_{eq}}}$$

However T_m might need to be limited bc of traction

Max Traction = Traction front ⊕ Traction back

* Since our system is 4WD (rigid connections)

→ Torque will go to where there is traction

→ If one has more traction than the other:

→ The other will stay at it's traction limit and act as a free axle... the other wheel will pull it up in speed

↑

It won't slip bc no torque above it's max is applied!

(Advantage of 4WD)

Conclusions:

$$\text{Power out} = \frac{d}{dt}\left(\frac{1}{2} KE\right) = \text{Power}_{\text{traction}}$$

$$T_m W_m = F_f V + F_r V$$

$$T_m \cancel{W_m} = (F_f + F_r)\, \cancel{W_m} k_{sys}$$

$$T_{limit} = T_m = \mu (N_f + N_r)\, k_{sys}$$

$* \; N_f + N_r = mg = \frac{\text{total weight}}{2}$

so

$$T_m = \begin{cases} T_{potential} & \text{if } T_{potential} < T_{limit} \\ T_{limit} & \text{if } T_{potential} \geq T_{limit} \end{cases}$$

so

$$\alpha_m = \frac{T_m}{I_{eq}}$$ > a function of time

$$W_m = \int \alpha_m \, dt$$

$$V = k_{sys} W_m = k_{sys} \int \alpha_m \, dt$$